Q. $\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}$ मैट्रिक्स के आइगेनवैल्यू (eigenvalues) और आइगेनवेक्टर (eigenvector) ज्ञात करें|

Solution:

दिया गया है :

$$\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}$$

characteristic equation is [A – λI] = 0

जहाँ: A matrix है और λ eigenvalue है|

इस प्रकार

$$\begin{bmatrix}5-\lambda & 4\\1 & 2-\lambda\end{bmatrix}$$

$\therefore [\lambda^2 - \{\text{some of diagonal elements}\}\lambda + \{\text{some of diagonal minors}\} = 0]$

some of diagonal elements $= \begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}$ = 5+2=7.

some of diagonal minors $= \begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}$ = 10-4 = 6.

Then

$= \lambda^2 - 7\lambda + 6 = 0$ [∴ 6 का FACTOR निकालते है|.... 1,2,3,6]

$= \lambda^2 - \lambda - 6\lambda + 6 = 0$

$= \lambda(\lambda - 1) - 6(\lambda - 1) = 0$

$= \lambda = 1, \lambda = 6$

इस प्रकार Eigenvalue 6 और 1 हैं।

Eigenvector:

[A – λI]x = 0

इस प्रकार

$$= \begin{bmatrix}5-\lambda & 4\\1 & 2-\lambda\end{bmatrix} \begin{bmatrix}x\\y\end{bmatrix} = 0$$

$\boldsymbol{\lambda = 6}$

$$= \begin{bmatrix} 5-6 & 4 \\ 1 & 2-6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$= \begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\frac{x}{y} = \frac{y}{1}$$

independent equation $-x + 4y = 0$

eigenvector is $(4, 1)$

$\boldsymbol{\lambda = 1}$

$$= \begin{bmatrix} 5-1 & 4 \\ 1 & 2-1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$= \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\frac{x}{1} = \frac{y}{-1}$$

eigenvector is (1, – 1)