Eigenvalues & Eigenvectors

Q. दिए गए मैट्रिक्स के आइगेनवैल्यू (eigenvalues) और आइगेनवेक्टर (eigenvector) ज्ञात करें|

$$\begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$$

Solution:

दिया गया है :

$$\begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$$

characteristic equation is [A – λI] = 0

जहाँ: A matrix है और λ eigenvalue है|

इस प्रकार

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$\therefore [\lambda^3 - \{\text{some of diagonal elements}\}\lambda^2 + \{\text{some of diagonal minors}\}\lambda - [A] = 0]$

some of diagonal elements $= \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$ = 8+7+3=18.

some of diagonal minors = $8-\lambda = \begin{bmatrix} 7 & -4 \\ -4 & 3 \end{bmatrix}$ = 21-16 = 5.

$7-\lambda = \begin{bmatrix} 8 & 2 \\ 2 & 3 \end{bmatrix}$ = 24 - 4 = 20.

$3-\lambda = \begin{bmatrix} 8 & -6 \\ -6 & 7 \end{bmatrix}$ = 56-36 = 20.

Total = 45.

Then

$= \lambda^3 - 18\lambda^2 + 45\lambda = 0$

$= \lambda(\lambda^2 - 18\lambda + 45) = 0$

$= \lambda(\lambda^2 - 15\lambda - 3 + 45) = 0$

$\therefore \lambda = 0$

इस प्रकार

$\lambda^2 - 15\lambda - 3\lambda + 45 = 0$

$\lambda(\lambda - 15) - 3(\lambda - 15) = 0$

इस प्रकार A के eigenvalues 0,3 और 15 हैं|

Eigenvector:

[A – λI]x = 0

इस प्रकार

$$= \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$\boldsymbol{\lambda = 0}$ मान रखने पर

$$= \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix} \begin{vmatrix} x \\ x_2 \\ x_3 \end{vmatrix} = 0$$

$$8x - 6x_2 + 2x_3 = 0 \quad (1)$$

$$-6x + 7x_2 - 4x_3 = 0 \quad (2)$$

$$2x - 4x_2 + 3x_3 = 0 \quad (3)$$

Use crammers rules from *eq.* (2) and (3)

$$x = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} = 21 - (-16) = 5.$$

$$-x_2 = \begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix} = -18 - (-8) = -10.$$

$$x_3 = \begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix} = 24 - 14 = 10.$$

$$x = 5, -x_2 = -10, x_3 = 10$$

इस प्रकार

$$= \begin{vmatrix} 5 \\ 10 \\ 10 \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ 2 \end{vmatrix}.$$

$\boldsymbol{\lambda = 3}$ मान रखने पर

$$= \begin{vmatrix} 8-3 & -6 & 2 \\ -6 & 7-3 & -4 \\ 2 & -4 & 3-3 \end{vmatrix} \begin{vmatrix} x \\ x_2 \\ x_3 \end{vmatrix} = 0$$

$$= \begin{vmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{vmatrix} \begin{vmatrix} x \\ x_2 \\ x_3 \end{vmatrix} = 0$$

$5x - 6x_2 + 2x_3 = 0$ (1)

$-6x + 4x_2 - 4x_3 = 0$ (2)

$2x - 4x_2 + 0x_3 = 0$ (3)

Use crammers rules from *eq.* (1) and (2)

$$x = \begin{vmatrix} -6 & 2 \\ 4 & -4 \end{vmatrix} = 24 - 8 = 16.$$

$$-x_2 = \begin{vmatrix} 5 & 2 \\ -6 & -4 \end{vmatrix} = -20 - (-12) = -8.$$

$$x_3 = \begin{vmatrix} 5 & -6 \\ -6 & 4 \end{vmatrix} = 20 - 36 = -16.$$

$x = 16, -x_2 = -8, x_3 = -16$

इस प्रकार

$$= \begin{vmatrix} 16 \\ 8 \\ -16 \end{vmatrix} = \begin{vmatrix} 2 \\ 1 \\ -2 \end{vmatrix}.$$

$\boldsymbol{\lambda = 15}$ मान रखने पर

$$= \begin{vmatrix} 8-15 & -6 & 2 \\ -6 & 7-15 & -4 \\ 2 & -4 & 3-15 \end{vmatrix} \begin{vmatrix} x \\ x_2 \\ x_3 \end{vmatrix} = 0$$

$$= \begin{vmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{vmatrix} \begin{vmatrix} x \\ x_2 \\ x_3 \end{vmatrix} = 0$$

$-7x - 6x_2 + 2x_3 = 0$

$-6x - 8x_2 - 4x_3 = 0$

$2x - 4x_2 - 12x_3 = 0$

Use crammers rules from *eq.* (1) and (3)

$$x = \begin{vmatrix} -6 & 2 \\ -4 & -12 \end{vmatrix} = 72 + 8 = 80.$$

$$-x_2 = \begin{vmatrix} -7 & 2 \\ 2 & -12 \end{vmatrix} = 84 - 4 = 80.$$

$$x_3 = \begin{vmatrix} -7 & -6 \\ 2 & -4 \end{vmatrix} = 28 + 12 = 40.$$

$x = 80, -x_2 = 80, x_3 = 40$

इस प्रकार

$$= \begin{vmatrix} 80 \\ -80 \\ 40 \end{vmatrix} = \begin{vmatrix} 2 \\ -2 \\ 1 \end{vmatrix}.$$

आइगेनवैल्यू (eigenvalues) = 0,3,15.

और आइगेनवेक्टर (eigenvector) = (1,2,2) , (2,1,-2) , (2,2,1),